



## Lyncrest Primary School

### Calculation Policy EYFS- Year 6

This calculation policy explains the teaching and learning progression of the four operations of addition, subtraction, multiplication and division. All these operations are taught in stages of concrete, pictorial and abstract methods to secure the children's understanding. Stages of calculation (below) do not refer to any particular year group as the methods taught are taught in relation to the developmental stage of individual children. Furthermore, methods from previous stages may be revisited at any time to deepen a child's understanding when solving problems, particularly the Singapore bar model.

Our overall aim in teaching calculation is that by the end of Year 6 all children will:

- have a secure knowledge of number facts and a good understanding of the four operations;
- be able to use their knowledge and understanding of number facts and the four operations to carry out mental and written calculations;
- make use of diagrams and informal notes to help record steps and part answers when using mental and written methods;
- have an efficient, reliable, compact written method of calculation for each operation that can be applied with confidence to calculations that cannot be performed mentally;
- use a calculator effectively, when required, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

#### Models and Images

It is important for children to have experienced a range of models and images to ensure that they develop fluency and a secure understanding of place value and the four operations. As such, across the teaching of all operations a variety of models and images are used



#### Contents

- Page 1-4 Addition
- Page 5-10 Subtraction
- Pages 11- 17 Multiplication
- Pages 18-26 Division

#### Addition

##### Stage 1: Concrete

Children use real life objects and apparatus to explore the different models of addition. This can be done in two ways:

1. **Augmentation:** two groups are combined; for example,

*There are 3 footballs in the red basket and 2 footballs in the blue basket. How many footballs are there altogether?*

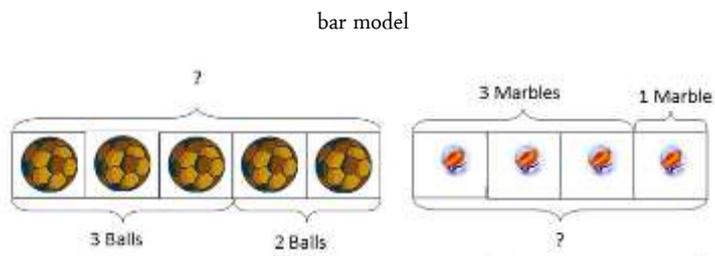
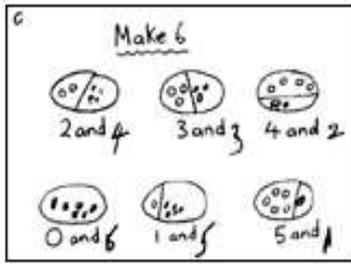
2. **Aggregation:** one group is added to; for example,

*Peter has 3 marbles. Harry gives Peter 1 more marble. How many marbles does Peter have now?*



##### Stage 2: Pictures

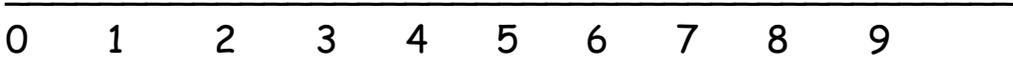
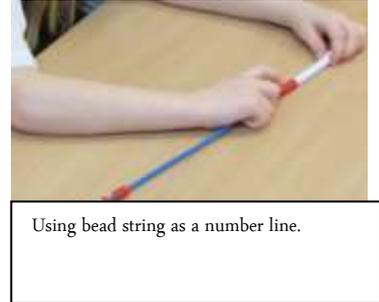
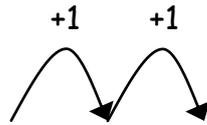
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures. Again **augmentation** and **aggregation** are utilised.



**Stage 3: The number line**

Children use number lines and practical resources to support calculation.

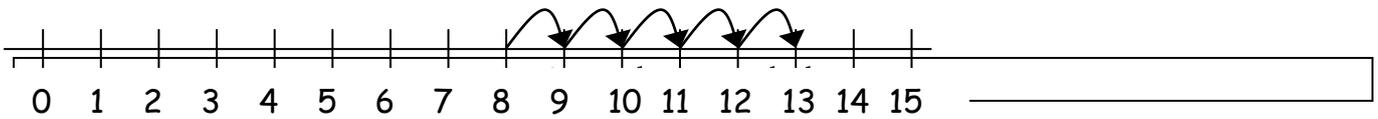
$$3 + 2 = 5$$



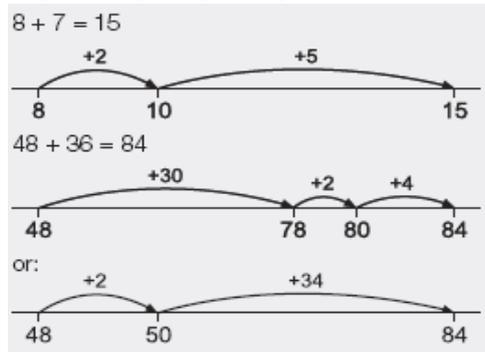
This will initially be done by counting on in ones.

$$8 + 5 = 13$$

+1 +1 +1 +1 +1



Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.

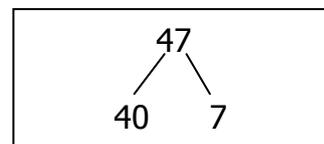


**Stage 5: Partitioning**

Children should be able to partition 2 and 3-digit numbers before adding them together.

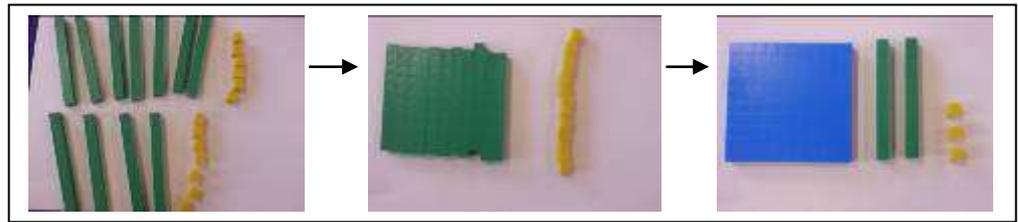
Some children find it useful to record partitioning in this way:

This partitioning method to the right shows a 'part, part, whole' model, where two parts make the whole. It also indicates what would be left if a part was taken away from a whole; for example if 7 was taken from 47 we would be left with only part of it, this being 40. A 'part, part whole picture' like this may be used at an earlier pictorial stage also. Further information about the 'part, part, whole' model can be found at the end of the addition and subtraction section.



At this stage, when adding two and three digit numbers, ones, tens and hundreds are partitioned and added separately to form partial sums and then these partial sums are added also to create a final sum. For example,

$$\begin{array}{r}
 47 + 76 = \\
 7 + 6 = 13 \\
 40 + 70 = \underline{110} \\
 \phantom{40 + 70} 123
 \end{array}$$

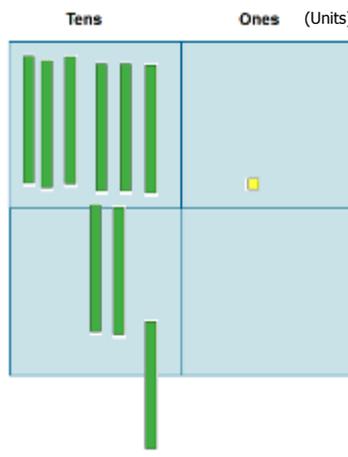
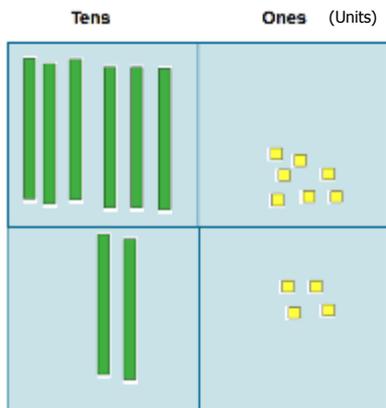


**Stage 6: Vertical Column method**

In this method, recording is reduced further. We aim for all children to become secure in this method as it is the most efficient written method. Regrouped digits are recorded below the line, using the words 'exchange and regroup ten' or 'exchange and regroup one hundred'. (The term 'carry' is not used)

This stage should be modelled with Base 10 apparatus first (shown in the example below through pictorial representation) or any other appropriate place value equipment (e.g. place value discs).

$$67 + 24$$



Exchange and regroup ten 1s for a 10.

$$\begin{array}{r}
 \text{TU} \\
 67 \\
 + 24 \\
 \hline
 91 \\
 \hline
 1
 \end{array}$$

Children can extend this method with any number of digits; for example,

$$\begin{array}{r}
 \text{HTU} \\
 783 \\
 + 42 \\
 \hline
 825 \\
 \hline
 1
 \end{array}$$

$$\begin{array}{r}
 \text{HTU} \\
 367 \\
 + 85 \\
 \hline
 452 \\
 \hline
 11
 \end{array}$$

$$\begin{array}{r} \text{ThHTU} \\ 7648 \\ + 1486 \\ \hline 9134 \\ \hline 1 \ 11 \end{array}$$

$$\begin{array}{r} \text{ThHTU} \\ 6584 \\ + 5848 \\ \hline 12432 \\ \hline 1 \ 11 \end{array}$$

$$\begin{array}{r} \text{ThHTU} \\ 42 \\ 6432 \\ 786 \\ 3 \\ + 4681 \\ \hline 11944 \\ \hline 1 \ 2 \ 1 \end{array}$$

Column addition remains efficient when used with larger whole numbers and decimals, provided that all digits are kept in the correct column for their place.

### Representing addition using the 'part, part, whole model' and 'bar model'

Part, part whole models are used across the year groups and in a variety of contexts to show in addition that the aggregate of parts make a whole amount. This can also be represented in a bar model. These two models help to secure understanding in the inverse relationship between addition and subtraction. For example, in the picture opposite, if you were to remove the part of 21 from the whole of 38, you would be left with the remaining part of 17.

The use of the bar model is particularly encouraged when visualising and solving word problems involving addition. In the picture below, it shows how two parts (£15.00 and £17.00 more than £15.00) can be pictorially compared to another part (£15.00)

part, part, whole

bar model

Bill has £15.00. Judy has £17.00 more. How much do they have altogether?

## Subtraction

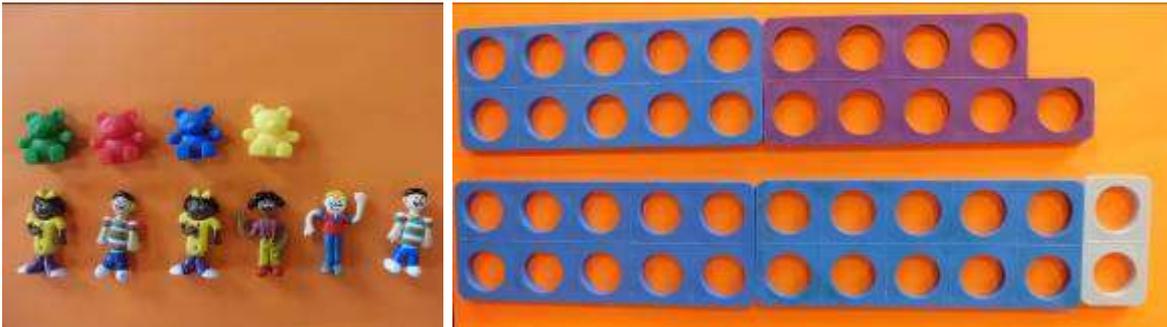
### Stage 1: Concrete

Children use real life objects and apparatus to explore two different models of subtraction:

1. Removing items from a set (reduction or taking away):



2. Comparing two sets (comparison or difference):



**Stage 2: Pictures**

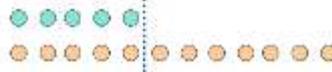
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures; for example,



**Removing items from a set (reduction or take-away)**

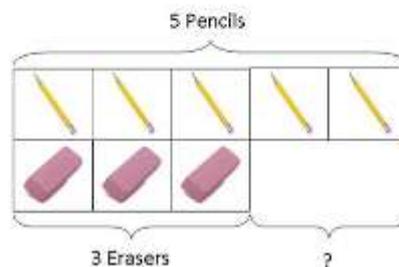


**Comparing two sets (comparison or difference)**



Singapore Bar Model:

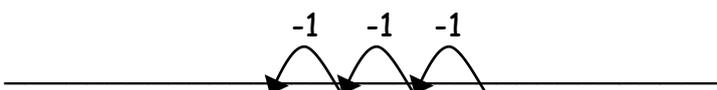
- Peter has 5 pencils and 3 erasers. How many more pencils than erasers does he have?



**Stage 3: The number line**

Children use number lines and practical resources to support calculation.

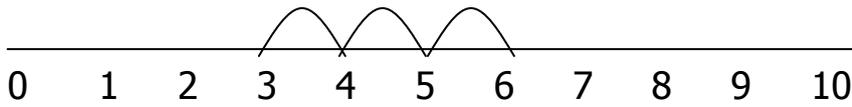
$$6 - 3 = 3$$



Using bead string as a number line.

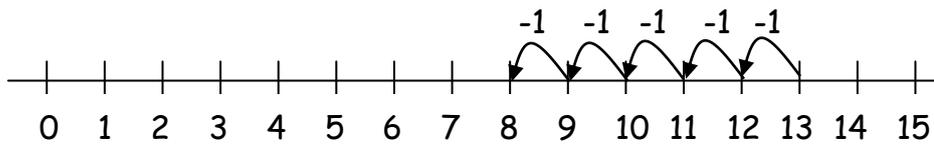
0 1 2 3 4 5 6 7 8 9 10

The number line should also be used to show that  $6 - 3$  means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many whole jumps they are apart.



Children can then begin to use numbered lines to support their own calculations using a numbered line to count back in ones to find difference. (This can also be developed to count forwards in ones to find the difference- as outlined in stage 4.)

$13 - 5 = 8$



**Stage 4: The empty number line**

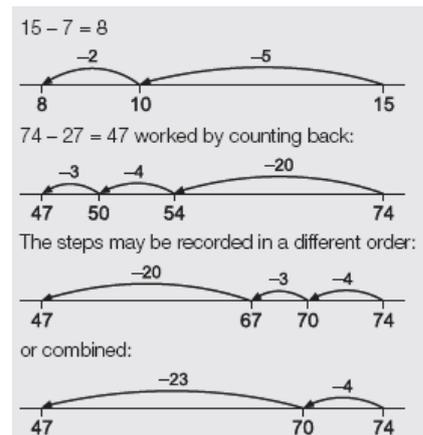
The empty number line helps to record or explain the steps in mental subtraction. A calculation like  $74 - 27$  can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.

**Counting backwards on the empty number line**

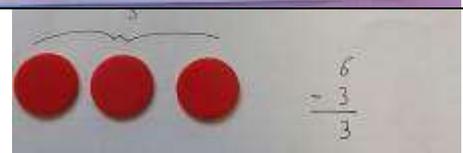
During the early stages of subtraction, counting backwards on a number line is the recommended strategy. However, it is expected that by the beginning of Key Stage 2 children have developed a concept of subtraction as finding the difference and should be encouraged to count forwards on a number line also.

**Finding the difference on the empty number line**

It is important to ensure that children understand why subtraction can be seen as finding the difference and why it makes sense to count up on a number line. An example of this is shown in the steps below.



<p>1. For <math>24 - 8</math>, find 24 on a number line</p>	
<p>2. Physically take away the 8 or clearly mark the 8 as being removed</p>	
<p>3. This is what is left. How can we work out how much this is worth? Count up from 8 to 24.</p>	



Finding the difference can be used by comparing amounts of objects also.

Finding the difference on a number line can be used with larger numbers, counting on between whole multiples of 10, 100, 1000 and decimal numbers etc...

<p>or:</p>	<p>or:</p>	<p>or:</p>
------------	------------	------------

**Stage 5: partitioning**

Subtraction can be recorded using partitioning to write equivalent calculations with the aim of this to be carried out mentally. For example,  $74 - 27$  involves partitioning the 27 into 20 and 7, and then subtracting the 20 and then the 7 in turn.

$$74 - 20 = 54$$

$$54 - 7 = 47$$

So  $74 - 27 = 47$

Recording partitioning can be linked to counting back (or forward) on the number line, as shown below.



**Stage 6: vertical column method**

In this method, recording is reduced further. We aim for all children to become secure in this method as it is the most efficient written method. Exchanged and regrouped digits are recorded above the whole and original numbers digits, using the words 'exchange and regroup ten' or 'exchange and regroup one hundred' etc.... (the term 'borrow' is not used) to explain the process.

This process is modelled with Base 10 first and in doing so exchanging and regrouping can be practised physically:

initial layout	exchanged and regrouped layout
----------------	--------------------------------

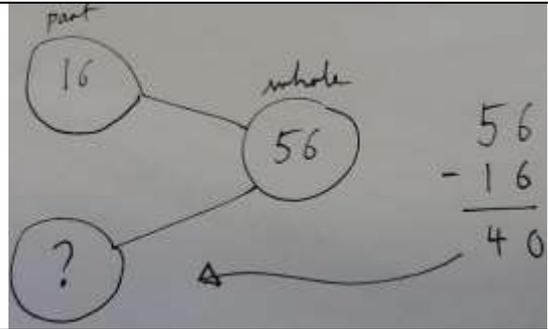
Once children are secure with exchanging and regrouping using base 10, and alongside this can record effectively, they may record their work solely in abstract form using this vertical column method. This can be extended to use with decimal numbers also, provided all digits are positioned in the correct place (see examples on the next page).

**Representing subtraction using the 'Part, part, whole model' and 'bar model'**

Part, part whole models are used across the year groups and in a variety of contexts to show in addition that the aggregate of parts make a whole amount. This can also be represented in a bar model.

These two models help to secure understanding in the inverse relationship between addition and subtraction. For example, in the picture opposite, if you were to remove the part of 21 from the whole of 38, you would be left with the remaining part of 17.

To find out a missing part you would simply subtract the part you know from the whole.



The use of the bar model is particularly encouraged when visualising and solving word problems involving subtraction. This can be used for reducing or separating a part from a whole (seen in the example in question 1 below).

Question 1: John takes 350 flowers to market. He sells 230. How many flowers does he have left?

A bar model representing the problem. A horizontal bar is divided into two sections. The top section is labeled "350" and the bottom section is labeled "230". The right portion of the bottom section is bracketed and labeled with a question mark "?".

Next to the bar model is a subtraction problem: 
$$\begin{array}{r} 350 \\ - 230 \\ \hline 120 \end{array}$$

Answer: 120 flowers

The bar model can also be used for finding the difference between two whole amounts using a comparison model (seen in the example in question 2 below).

Question 2: Bobby has 47 cards. Jules has 127 cards. How many more cards does Jules have?

A comparison bar model. Bobby's bar is labeled "47" and Jules's bar is labeled "127". A bracket above the bars indicates the difference between them, labeled with a question mark "?".

Next to the bar model is a subtraction problem: 
$$\begin{array}{r} 127 \\ - 47 \\ \hline 80 \end{array}$$

Answer: 80 cards.

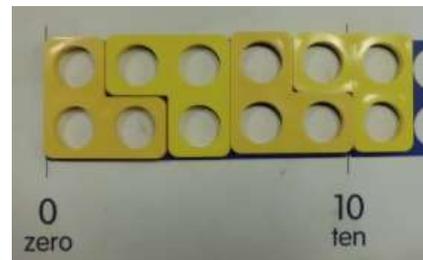
## Multiplication

### Stage 1: Concrete

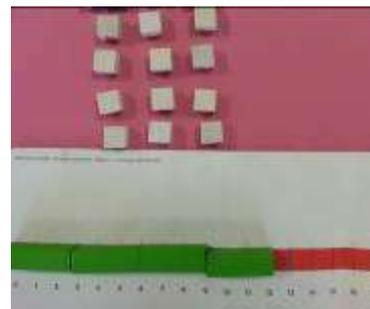
Children use real life objects and apparatus to explore the different models of multiplication; For example, 'lots' of or 'groups' of the same thing. This is essentially repeated addition.



Children can then work on placing these lots into arrays to recognise the **multiplicand** (the fixed amount to be multiplied), the **multiplier** (the amount of times the fixed number is repeatedly added) and the product (final amount). In each of the arrays to the right, the number sentence is  $4 \times 3 = 12$ . 3 is the fixed amount: the multiplicand. The Multiplicand of 3 is multiplied or repeatedly added four times. This makes a product of 12.



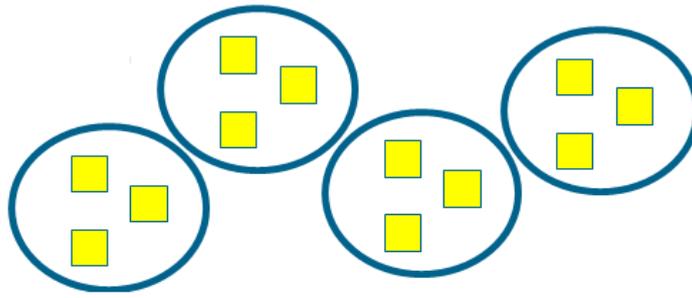
These arrays can be transferred to a number line to link concrete, pictorial and abstract understanding of multiplication (depending on the readiness of the children).



At all stages of learning (Year N to 6) there will be practise with concrete objects and using these concrete objects the idea of **scaling** will be introduced. This is where an item or amount scales up to become larger or scales down to become smaller. In the arrays to the right and above the amount of 3 becomes four times bigger. (This is at a ratio of 1:4. For every one cube there is four times as many.) Scaling is explained further in multiplication through pictures.

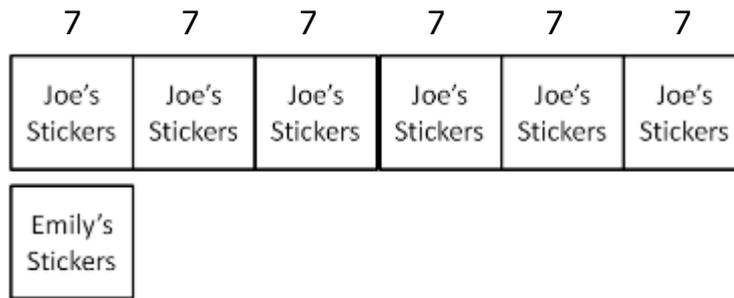
### Stage 2: Pictures

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures.



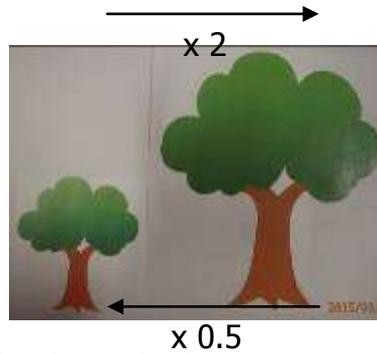
At the pictorial stage we also encourage the use of bar modelling to represent problems; for example,

'Emily has 7 stickers, Joe has six times as many stickers. How many Stickers does Joe Have?'



In the bar above, each part of the bar represents 7 stickers. Emily has one lot of 7. Joe has six lots of 7.

The children at the pictorial stage can also gain a further understanding of multiplication as scaling; for example, in the picture below, the tree on the right is 2 times bigger than the tree on the left.

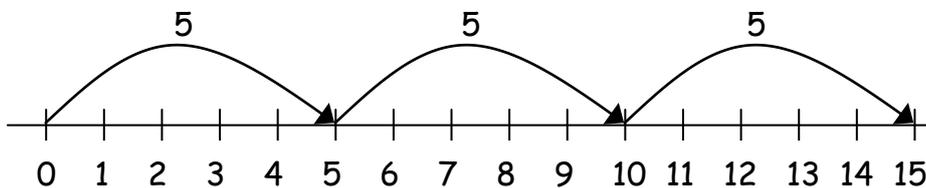


Inversely, the tree on the left is half (1/2 or 0.5 times) as big as the tree on the right.

**Stage 3: The number line**

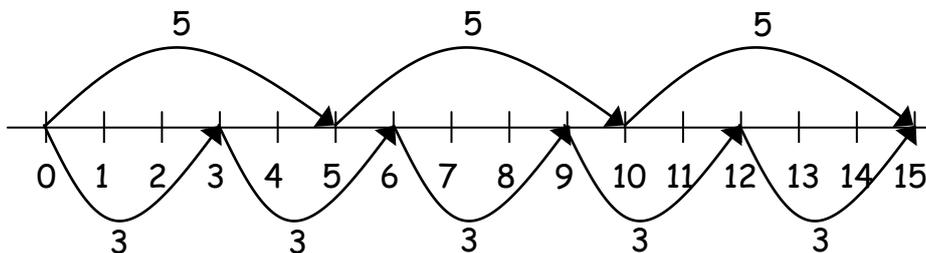
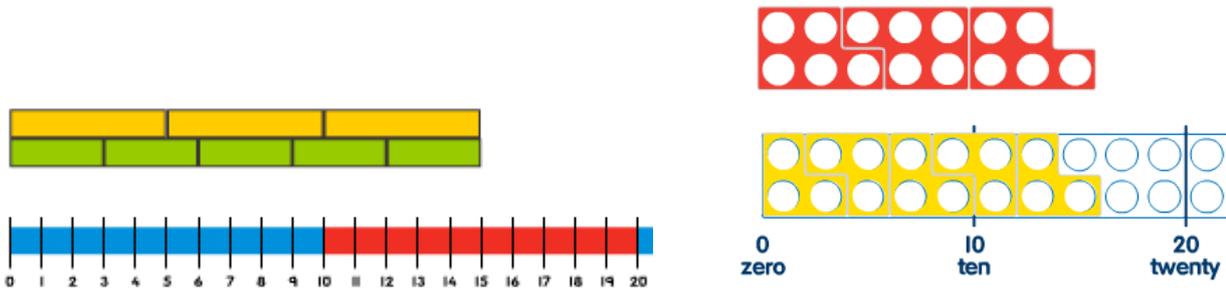
Multiplication on a number line is represented through repeated addition and can be shown easily on a number line:

3 times 5 is  $5 + 5 + 5 = 15$  or 3 lots of 5 or  $5 \times 3$



This repeated addition can also be referred to as **skip counting** - skipping past certain numbers on a number line.

Children should know that  $3 \times 5$  has the same product as  $5 \times 3$  (the commutative law of multiplication). This can also be shown on the number line.



When children record any findings at this stage, they will be encouraged to use the correct mathematical signs (+,  $\times$  and =) and be made aware that the product of any multiplication can come at any point within a number sentence; for example,

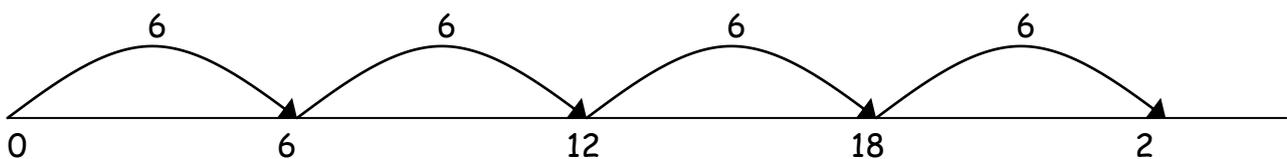
$$15 = 5 + 5 + 5 = 5 \times 3$$

Here, 15 (the product) is placed at the beginning of the number sentence.

**Stage 4: The empty number line**

Children will continue to use repeated addition, which can also be referred to as **skip counting**, but will progress onto using a blank number line:

4 times 6 is  $6 + 6 + 6 + 6 = 24$  or 4 lots of 6 or  $6 \times 4$



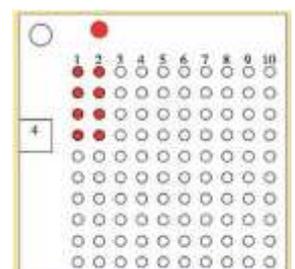
**Stage 5: Dot paper**

At this stage, the children will most likely be continuing to use a combination of concrete and pictorial representations as well as using number lines and skip counting; however, this stage can be done in tandem with other methods and reinforces visualising multiplication products in arrays.

To the right, the example represents four lots of two or

$$2 \times 4 = 2 + 2 + 2 + 2 = 8$$

At this stage children are visually reminded that the multiplicand is the fixed amount (in this example it is 2) and the multiplier is how many the fixed amount is multiplied by (in this example it is 4).



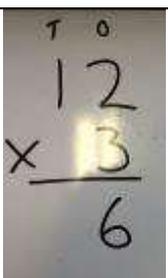
**Stage 6: Vertical column method with a one digit multiplier**

This stage comprises of four phases.

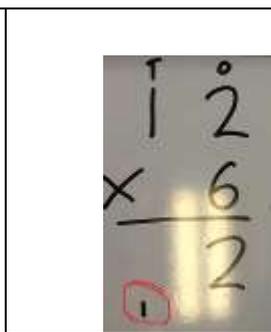
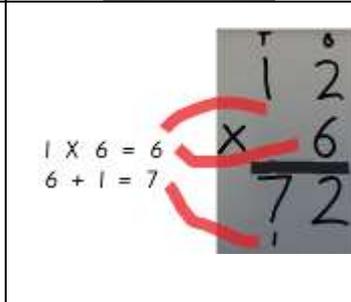
**Phase 1: multiplying by a one digit multiplier and without regrouping**

At this phase, concrete apparatus such as Base 10 or place value discs may and should be used to support learning.

Children begin the vertical column method by multiplying a two digit number by a one digit number where regrouping does not take place. This can then be extended further by multiplying three and four digit numbers by one digit numbers. Essentially, regrouping happens when the product of a calculation is larger than nine. This process involves partitioning the numbers being worked with and in the case of  $12 \times 3$ , undertaking these calculations:  $2 \times 3$  and  $10 \times 3$ .

<p>a) The ones are multiplied by the multiplier first and the children recognise they are calculating: <math>2 \text{ ones} \times 3 = 6 \text{ ones}</math>.</p>		<p>b) After the ones are multiplied, the tens are multiplied by the multiplier and the children recognise they are calculating: <math>1 \text{ ten} \times 3 = 3 \text{ tens}</math>.</p>
---	---	---

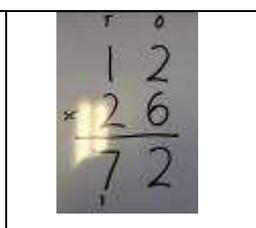
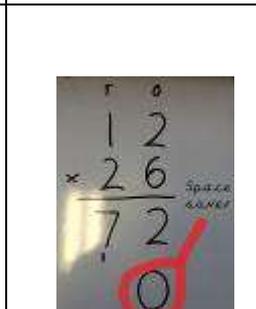
**Phase 2: multiplying by a one digit multiplier with regrouping**

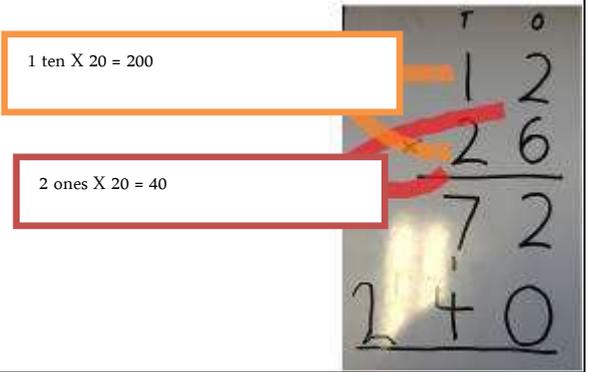
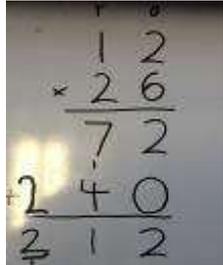
<p>Essentially, regrouping happens when the product of a calculation is larger than nine. As in the example below of <math>12 \times 6</math>, when the ones number is multiplied it makes 12 and therefore the ten in that product is regrouped into the tens column. Again, the child recognises that <math>2 \text{ ones} \times 6 = 12 \text{ ones}</math>. The regrouped ten (circled in red) is then added on to the product of the next calculation.</p> <p>Again also, in this calculation, the number sentence is partitioned into two sections: <math>6 \times 2</math> and <math>6 \times 10</math>.</p>	
<p>The next calculation in this example is <math>1 \text{ ten} \times 6 = 6 \text{ tens}</math>. However, because of the regrouped ten, the final product in the tens column is now 7 tens.</p> <p>This can be extended further by multiplying three and four digit numbers by one digit numbers</p>	

**Phase 3: multiplying by a two digit multiplier with regrouping**

Before moving onto this phase, children should have a secure understanding of place value and be able to multiply confidently by multiples of 10 and then 100, 1000 etc (The teaching of multiplying by 10, 100 or 1000 should not be taught through how many zeros are placed on the end of a number; children should understand that as the digit moves to the left on a place value grid, its value becomes ten times greater and the zero is a place holder. For example, 100 is ten times greater than 10.)

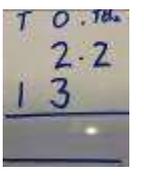
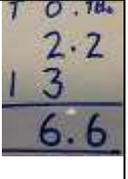
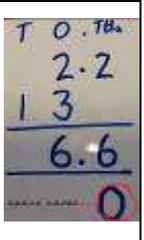
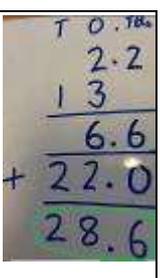
This phase takes place in the following steps:

<p>1. The multiplicand is multiplied by the ones digit, partitioning the multiplication of the ones and the tens. In this example we have <math>2 \times 6</math> and <math>10 \times 6</math> and the product is displayed below the line.</p>	
<p>2. Next, the multiplicand is multiplied by the tens number. Because we are multiplying by a tens number, every product will be ten times bigger and as such must be moved over one space. This is accomplished by using a 0 as a space saver. If you were multiplying by a hundreds number, you would use two 0's as space savers to make the products one hundred times bigger and similarly, if you were multiplying by a thousand number, you would use three 0's as space savers to make the products one thousand times bigger etc...</p>	

<p>3. Now, the multiplicand is multiplied by the digit in the tens column but because we have added a space saver we are also multiplying each time by ten and can essentially just calculate what each digit of the multiplicand is multiplied by according to the tens digit. Again, we partition the ones and tens and multiply the ones (the smallest unit) first.</p>	
<p>4. To complete the multiplication, the products from the ones multiplier and the tens multiplier are added together using vertical addition. This process can then be used for any digit number multiplied by two, three and four digit numbers where necessary.</p>	

**Phase 4: multiplying decimal multiplicands**

The vertical method can then be extended to multiplication of decimal numbers, providing the children have a secure understanding of place value, as it is imperative that when multiplying decimals, all numbers are kept in the correct place. This is explained in the steps below.

<p>1. All the digits are put in the correct place. This is visible in the layout of 2.2 X 13, where all digits are in the correct place.</p>	
<p>2. The 2.2 is multiplied by the ones number and the product of each multiplication is placed in the correct place.</p>	
<p>3. When multiplying by the tens number, a space saver is added in the tenths column (the smallest place of the multiplicand in this case) to ensure all digits in the product are 10 times bigger.</p>	
<p>4. The multiplicand is multiplied by the digit in the tens column but because we have added a space saver we are also multiplying each time by ten and can essentially just calculate what each digit of the multiplicand is multiplied by according to the tens digit. Again, we partition the tenths and ones and multiply the tenths (the smallest unit) first. Lastly, the products of the multiplication by the ones number and the tens number are added together to give the final product (circled in green)</p>	

## Representing multiplication using diagrams and the 'bar model'

The use of diagrams and bar models is particularly encouraged when visualising and solving word problems involving multiplication. Below is an example of a question where diagrams and a 'bar model' can be used to solve it.

**Question:** 'One container can hold  $450\text{cm}^3$  of orange juice. Mr Thomas has to fill five of these containers. The juice costs  $15\text{p}$  per  $\text{cm}^3$ . How much does it cost Mr Thomas to fill the five containers?'

To the right is an example of how this can be solved using diagrams and the bar model. Bar models can also be used when comparing amounts as can be seen in the question and solution below.

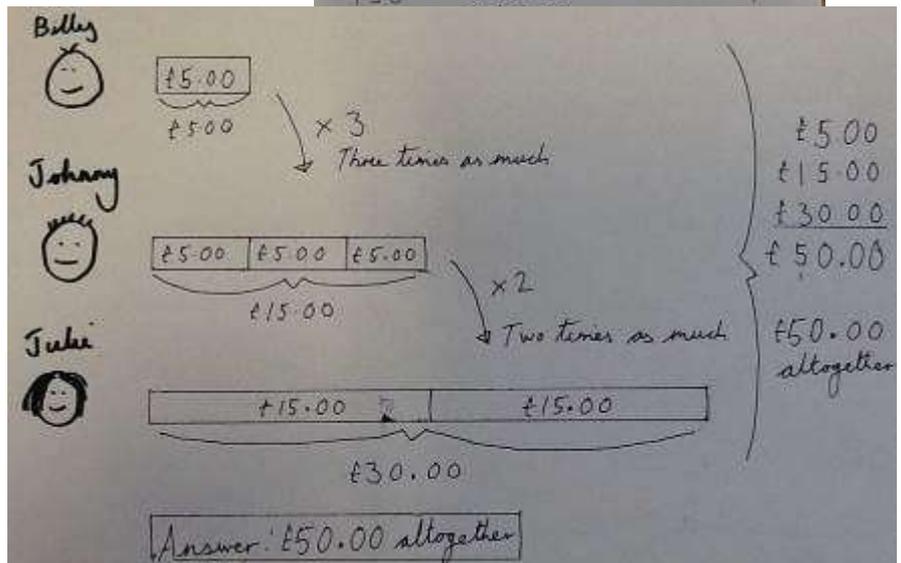
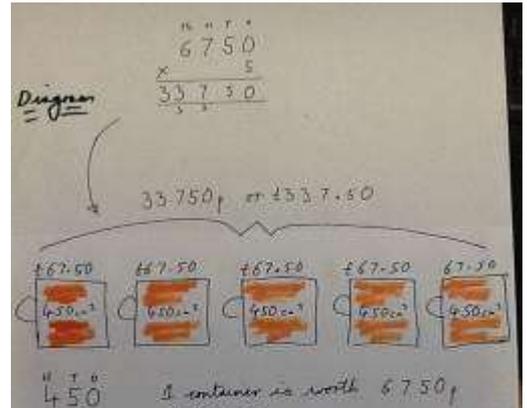
**Question:** 'Billy has  $\pounds 5.00$ . Johnny has three times as much as this. Julie has twice as much as Johnny. How much money do they have altogether?'

The solution to question (to the right), demonstrates how the bar model in this instance can be related to ratio. In this case, Billy has 1 part, Johnny has 3 parts and Julie has 6 parts. This as a ratio would be 1:3:6 making a total of 15 or, as an equivalent amount, 5:15:30 making a total of 50, as seen in the answer.

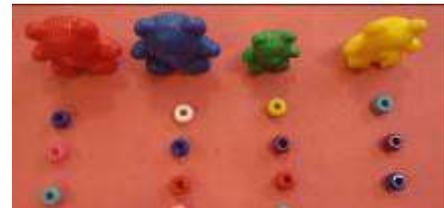
### Division

#### Stage 1: Concrete

Children use real life objects and apparatus to explore the different models of division and practise sharing objects or placing them into groups (lots of). It is important to distinguish the difference between **grouping** and **sharing** as children need to know which of these models is appropriate for a given problem.



In the **sharing model**, things are shared between other things. For example, in the picture to the right 20 beads are shared equally between 4 bears. This is  $20 \div 4 = 5$



In the **grouping model** an amount of things is put into groups or lots of. For example, in the picture on the right, 20 people are put into equal groups of 4 or equal lots of 4. This is  $20 \div 4 = 5$ .



A key idea in sharing or grouping is the idea that amounts are organised equally and anything left over is considered as a remainder.

#### Stage 2: Pictures

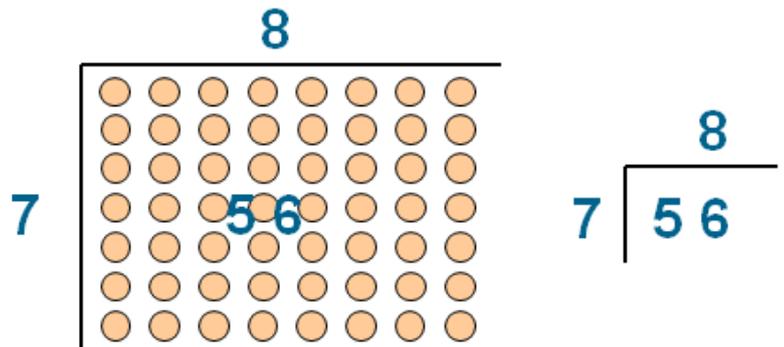
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures. Below are examples of pictorial division using grouping models (amounts are placed into groups).

$12 \div 4 = 3$ 3 groups/ lots of 4	$20 \div 5 = 4$ 4 groups/ lots of 5	$21 \div 5 = 4 \text{ r } 1$ 4 groups/ lots of 5 with one remaining

Division can also be represented pictorially in arrays and in this context (to the right), the children can see not only why division is laid out in a 'bus stop' style but they can also link division to the inverse operation of multiplication:

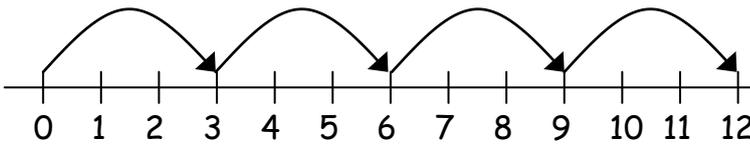
$$56 \div 8 = 7 \text{ and } 7 \times 8 = 56$$

$$56 \div 7 = 8 \text{ and } 8 \times 7 = 56$$



**Stage 3: The number line**

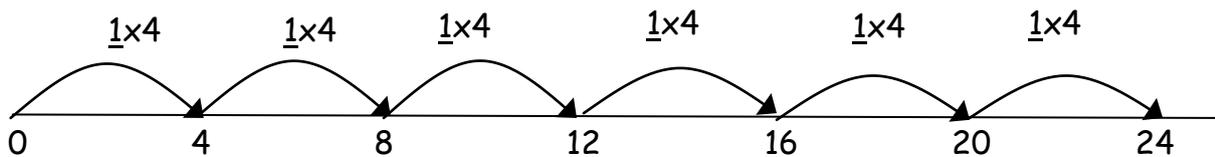
If it is helpful to the child's understanding, number lines can be used to see how many lots of amount are in a number. This is particularly useful when using Numicon resources (seen in the yellow lots of three below). The use of Numicon also links division back to a concrete experience of sharing and grouping. In the example below of  $12 \div 3 = 4$ , the children can count on a number line to see how many groups/ lots of 3 are in 12.



**Stage 4: The empty number line**

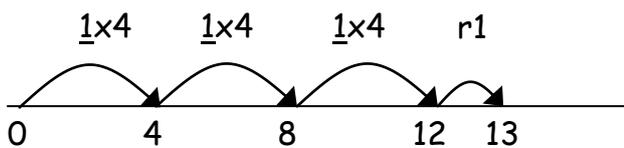
If it is helpful to the children's understanding, they can next progress onto using a blank number line. This can be seen below in  $24 \div 4$ : here it is useful to explain this as 'How many lots of 4 make 24?' or 'How many groups of 4 are in 24?'

$$24 \div 4 = 6$$



This empty number line method can also be used to show the next progression of division with remainders.

$$13 \div 4 = 3 \text{ r } 1$$

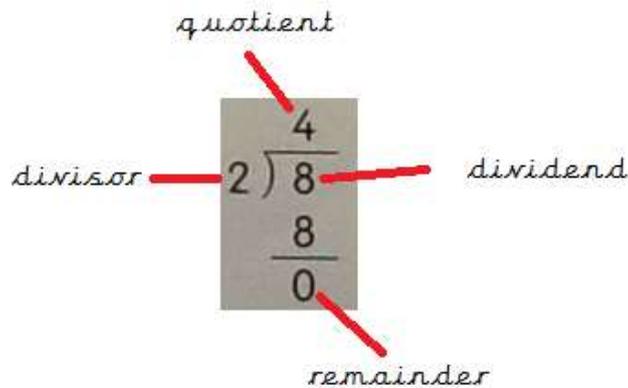


The empty number line should also be used to relate division to the inverse operation of multiplication, as it represents multiplication as skip counting.

**Stage 5: Long Division**

Generally, depending on the needs of the children, they will move from stage 1 and 2 in division (grouping and sharing concrete and pictorially) and go past stage 3 and 4 (division using number lines), and move onto Stage 5, recording division vertically using the long method but aided with pictorial support. Knowledge of all the multiplication and division facts are essential for this stage.

In the teaching of long division (and any other division method), the correct vocabulary must always be used (annotated in the diagram below).



The teaching of long division is comprised of six phases (outlined below)

**Phase 1: long division (of one and two digit numbers) with no regrouping and no remainders.**

At this phase, digits of place do not need to be broken into smaller values and no amounts are left over when grouping or sharing. This can be seen in the example of  $8 \div 2 = 4$ , which could be considered as 8 shared by 2 (represented below pictorially and with a vertical calculation).

'Bill and Jane are collecting water. They have eight buckets and share these buckets equally between them. How many buckets do they each have?'



$$\begin{array}{r} 4 \\ 2 \overline{) 8} \\ \underline{8} \\ 0 \end{array}$$

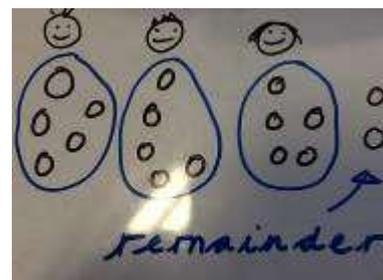
Whether or not a grouping model or a sharing model would need to be used for pictorial application would be based on the context of the question.

**Phase 2: long division (of one and two digit numbers) with no regrouping and remainders**

At this phase, digits of place do not need to be broken into smaller values; however, amounts are left over when grouping or sharing.

The example below is based on this question:

'3 children share 17 balls equally between them. How many balls does each child have? How many balls are left over?'



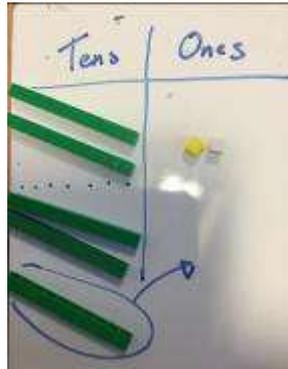
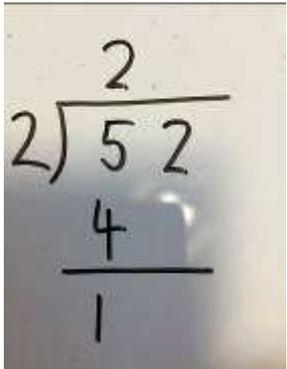
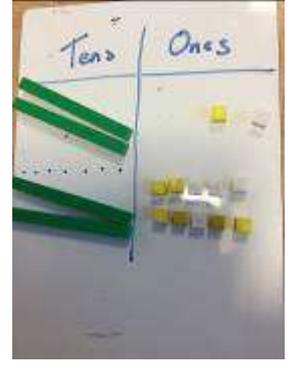
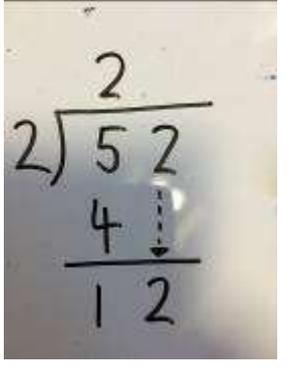
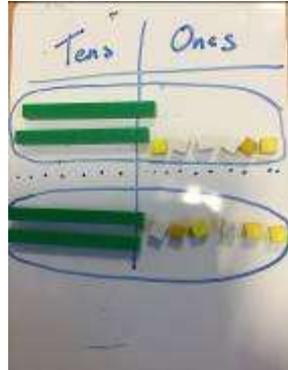
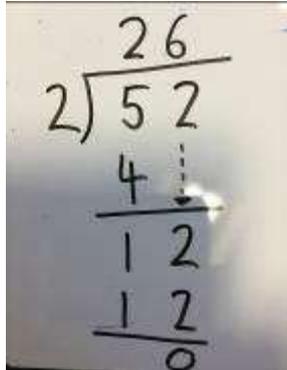
When using long division to calculate this, the dividend (17) is shared by the divisor (3) and the quotient of this (5) is recorded along the top (of what we may refer to as the 'bus stop'). The product of the divisor and the quotient (the product of 3 and 5, being 15) is then recorded under the dividend and subtracted from the dividend, giving the remainder (which in the example above is 2).

**Phase 3: long division (of one and two digit numbers) with remainders and regrouping.**

At this phase, digits of place need to be broken into smaller values and amounts are left over when grouping or sharing. During this phase, pictures or manipulatives should be used to secure the learning. Furthermore a recorded method is used in tandem with manipulatives, such as Base 10 or place value discs, until children can divide confidently without concrete aids.

The example below of this phase is based on this question:

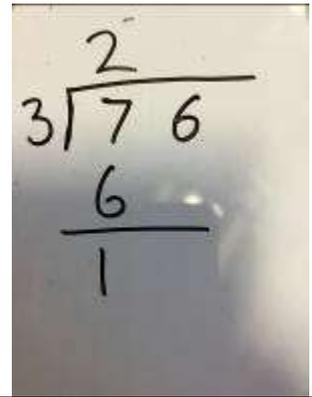
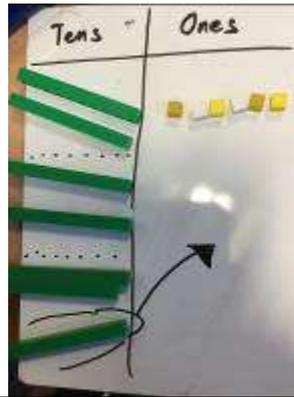
'Billy and Sade share 52 playing cards equally. How many playing cards does each person have?'

<p>1. The tens place is divided by two. This leaves one remainder of ten that is to be regrouped in the unit column.</p> <p>The product of 2 tens is 4 tens and this is recorded and taken away from the tens place (<math>5-4=1</math>) This 1 ten is recorded at the bottom of the written division.</p>	 
<p>2. The remaining ten is regrouped and changed for ones leaving twelve ones.</p> <p>On the written method the 2 digit in ones column is brought down to meet the 1 ten to make 12.</p> <p>Next, the 12 will be shared by 2.</p>	 
<p>3. The 12 ones are shared by two, leaving a total of 26 (cards) for each person.</p> <p>On the written method the product of 2 and 6 (12) is placed under the 12 and subtracted, leaving no remainder in this case.</p> <p>6 is placed in the ones section of the quotient because 12 divided by 2 is 6 ones.</p>	 

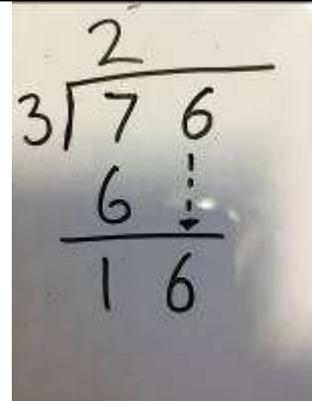
This next example has a final remainder and is based on the question below:

'James, Mia and Jaiya share 76 beads equally. How many beads does each child get?'

1. First, divide the tens by 3.  
7 tens divided by three equals two tens with one remainder

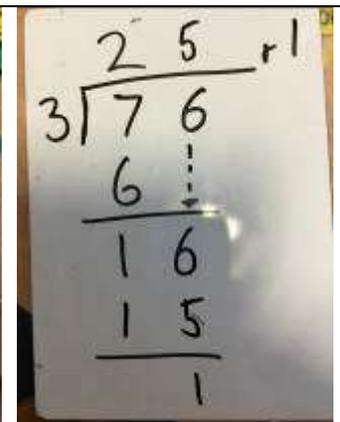
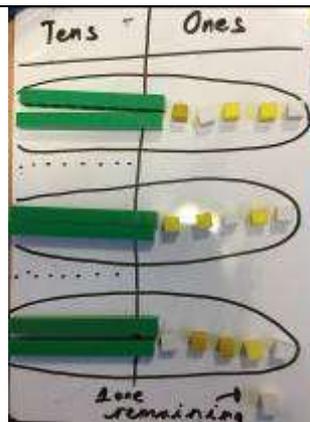


2. Next, regroup the remainder ten so that one ten is equal to ten ones. Add the tens and the ones creating 16 ones.



3. Divide the ones by 3. 16 ones divided by three is equal to 5 ones with a remainder of one.

This gives a final quotient/ answer of 25 (beads per person) and one remainder/ left over.



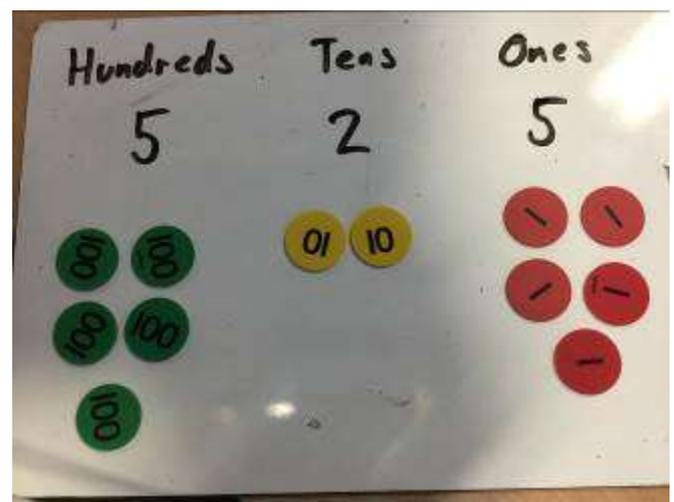
**Phase 4: long division of numbers with tens, ones and hundreds (three digit dividends)**

In this phase, the same process is followed; however, initially the children will work on problems that have three digit numbers and no remainders. For example:

$$525 \div 3 = 175$$

Next, the children will undertake problems with three digit numbers (as the dividend) and remainders. For example:

$$525 \div 4 = 131 \text{ r}1$$



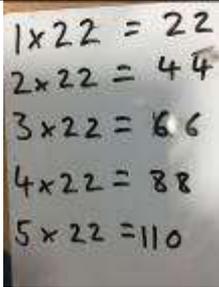
At this stage they may feel confident at completing these number sentences without manipulatives or they may use other representational manipulatives such as place value counters (pictured to the right)

The children can also, when confident begin to extend with four digit dividends featuring thousand columns (e.g. 1234)

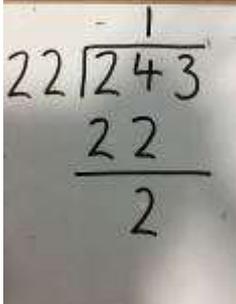
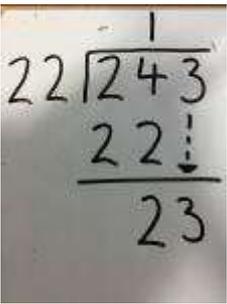
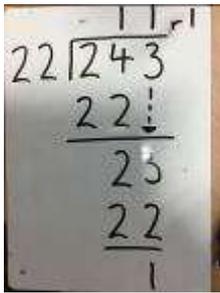
**Phase 5: division with a two digit divisor**

Due to the nature of the large amounts of sharing or grouping necessary (on account of the two digit divisor), children can only undertake this phase if they are secure with using the written long division in the previous phase and no longer need manipulatives.

Also, during this phase, it is helpful that the children create a multiplication table of the divisor. The steps for this phase are outlined below.

<p>1. For the problem of <math>243 \div 22</math> the first advisable stage is to create a multiplication table for the divisor of 22.</p>	
--	---

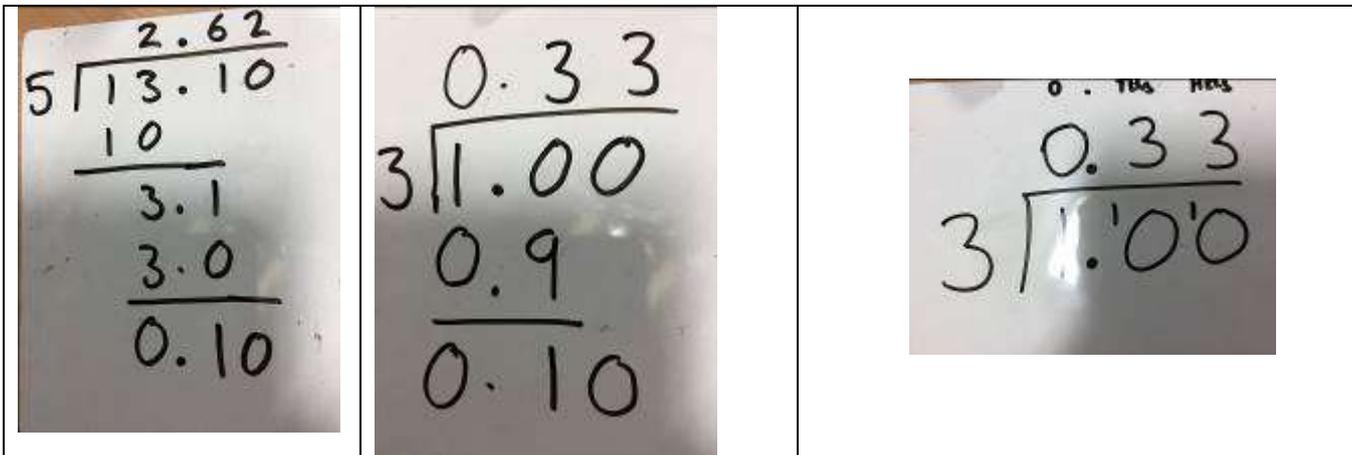
<p>2. The 2 hundreds need to be regrouped with the 4 tens, making 24 tens and this is divided by 22, leaving a remainder of 2 tens.</p>	<p>3. The 2 tens are regrouped with the 3 ones, making 23 ones.</p>	<p>4. The 23 ones are shared by 22, giving a final quotient of 11 with a remainder of 1.</p>
---	---	--

		
---	---	---

**Phase 6: long division with a decimal dividend and decimal remainders**

In this last phase, the same written long division process is followed; however, when undertaking a problem working to three decimal points and rounding to two decimal places is advised (as money uses two decimal points) and place savers of zero are added into any unpopulated tenths, hundredths and thousandths spaces. To be successful at this phase, a secure understanding of place value is essential.

<p><math>13.10 \div 5</math> A zero is added in the hundredth space of the dividend also.</p>	<p><math>1 \div 3</math> A zero is added into the tenth and hundredth space to create spaces for a decimal remainder. Here the 3 in the quotient is recurring (creating a quotient of 0.333333...) but only two decimal places are used.</p>	<p><math>1 \div 3</math> This is the same calculation as the previous one only this is represented in a compact short division method. This method is only taught when children's knowledge of multiplication and division facts and their understanding of partitioning and place value is totally secure.</p>
---	--	---



**Representing division using diagrams and the 'bar model'**

The use of diagrams and bar models is particularly encouraged when visualising and solving word problems involving division. Diagrammatic and pictorial models are particularly encouraged early on when undertaking division; however, in the upper key stages bar modelling can become essential to solving more complex problems. For example:

Joe has 6 times as many stickers as Emily. Joe has 48 stickers. How many stickers does Emily have?

We know that Joe has 48 stickers in total.

The model shows us that we must calculate  $48 \div 6$

Joe has 6 times as many stickers as Emily. Joe has 48 stickers. How many stickers does Emily have?

It is clear then that Emily also has 8 stickers...

$48 \div 6 = 8$

The bar model is particularly effective when combined with a relevant diagram as shown when answering this question, outlined in the steps below:

'A pile of 5 identical books, when stacked face to face, has a height of 36cm. If two more books were added to the pile, what would the height then be?'

<p>1. Draw a diagram of the book stack and divide its total height by the amount of books.</p>	<p>2. Represent the book stack in a bar and highlight (with a question mark) the section that is missing.</p>	<p>3. Add all sections of the bar to find the final answer.</p>